BUDDHA SERIES

(Unit Wise Solved Question & Answers)

Course – B.Tech

College – Buddha Institute of Technology

Department: Civil Engineering

Subject: Structural Analysis (BCE - 502)

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Unit - 1

Q1. Explain in detail the classification of structures.

Answer:
Structures are the load-resisting systems that transfer loads safely to the ground. They can be classified into the following categories:

1. **Based on Form/Geometry:**
    - **Line Structures:** One-dimensional load-carrying members like beams, trusses, cables.
    - **Surface Structures:** Two-dimensional such as slabs, plates, shells.
    - **Space Structures:** Three-dimensional load-carrying systems like domes, grids, space frames.
2. **Based on Load Transfer Mechanism:**
    - **Truss Structures:** Carry axial forces only (tension or compression). Example: roof trusses.
    - **Rigid Frames:** Carry axial, shear, and bending moment. Example: multi-storey frames.
    - **Arches:** Carry mainly compression forces; economical in masonry construction.
    - **Cables & Suspension Bridges:** Carry only tensile forces; suitable for long spans.
3. **Based on Materials Used:**
    - Timber structures, Steel structures, RCC (Reinforced Cement Concrete), and Composite structures.
4. **Based on Function:**
    - Building structures, Bridge structures, Tower structures, Hydraulic structures, etc.

Conclusion: Classification helps engineers choose the most economical and efficient structure for a given function.

Q2. Discuss different types of structural frameworks and explain their load transfer mechanisms.

Answer:
Structural frameworks are systems of members arranged to resist loads. The types and their mechanisms:

1. **Beams:**
    - Horizontal members carrying loads mainly by *bending*.
    - Transfer loads through shear force and bending moment to supports.
2. **Trusses:**
    - Formed by straight members connected at joints.
    - Loads transferred as *axial forces*.
    - Very efficient in covering large spans with minimal material.

3. **Rigid Frames:**
    - Consist of beams and columns rigidly connected.
    - Carry bending moment, shear, and axial force.
    - Common in multi-storey buildings.
4. **Arches:**
    - Curved members mainly subjected to compression.
    - Efficient for bridges and aqueducts.
5. **Cables:**
    - Flexible members carrying loads in *tension only*.
    - Adopt shapes depending on applied load (polygonal/parabolic).

Conclusion: Understanding the load transfer mechanism ensures efficient structural design and safety.

Q3. Define stress resultants. Explain with reference to beams.

Answer:

- Stress resultants are the *internal forces and moments* that develop in structural members to balance external loads.

For **beams**, the main stress resultants are:

1. **Axial Force (N):** Internal force along the longitudinal axis.
2. **Shear Force (V):** Internal force perpendicular to axis, responsible for shear stresses.
3. **Bending Moment (M):** Internal couple causing bending stresses.

Equations:

- From equilibrium,
  $$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$
  give the values of N, V, and M.

Example:

- Simply supported beam under UDL:
    - Shear force varies linearly.
    - Bending moment varies parabolically, max at midspan.

Conclusion: Stress resultants form the basis for design, ensuring members are safe against failure.

Q4. Explain the concept of degrees of freedom (DOF) in structural systems with examples.

Answer:

- **Definition:** Degree of Freedom is the minimum number of independent displacements required to define the position of a structure.

Types of DOF in 2D:

- **Translation in x-direction**
- **Translation in y-direction**
- **Rotation about z-axis**

Examples:

- Pin-jointed joint → 2 DOFs (translations).
- Rigid joint in frame → 3 DOFs (2 translations + 1 rotation).
- Simple beam element in 2D → 2 nodes × 2 DOFs = 4 DOFs.

Importance:

- DOFs are the basis for matrix methods (stiffness and flexibility methods).
- Directly related to **Kinematic Indeterminacy**.

Conclusion: Knowing DOF is essential for formulating equations of motion and structural analysis.

Q5. Differentiate between static indeterminacy and kinematic indeterminacy. Give examples.

Answer:

Aspect	Static Indeterminacy (SI)	Kinematic Indeterminacy (KI)
Definition	Extra unknown forces than equilibrium equations	Number of unknown displacements
Formula	For truss: SI=m+r−2jSI = m + r - 2jSI=m+r−2j	Count of DOFs in system
Example	Fixed beam (SI=1)	Two-span continuous beam (KI=2)

Explanation:

- **SI** arises from redundant supports or members.
- **KI** arises from possible independent joint displacements.

Conclusion: Both are equally important — SI for force method, KI for displacement method.

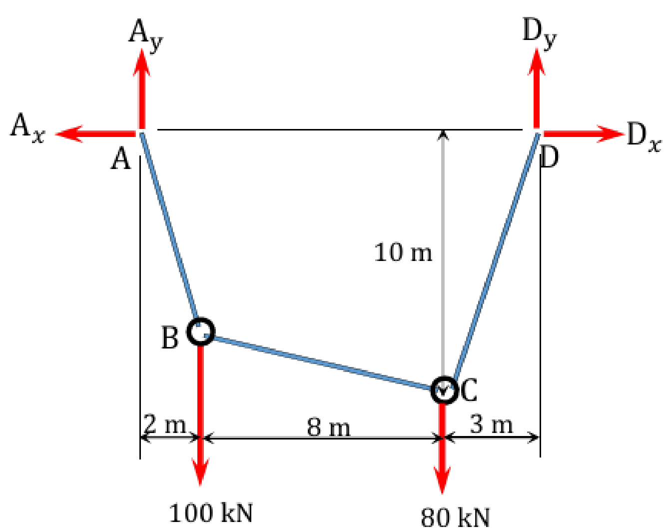
Q6. Explain the procedure to calculate static indeterminacy of beams, trusses, and frames.

Answer:

1. **Beams (2D):**
   SI=r−2SI = r - 2SI=r−2, where *r* = number of support reactions.
   - Example: Fixed beam (r=3) → SI = 1.
2. **Trusses (2D):**
   SI=m+r−2jSI = m + r - 2jSI=m+r−2j, where *m* = members, *j* = joints, *r* = reactions.
   - Example: Truss with m=9, j=5, r=3 → SI = 2.
3. **Rigid Frames (2D):**
   SI=(m′+r)−3jSI = (m' + r) - 3jSI=(m′+r)−3j, where *m′* = total member forces.

Conclusion: Knowing SI helps decide whether a structure can be solved by equilibrium alone or requires compatibility equations.

Q7. A cable supports two concentrated loads at B and C, as shown in Figure 6.8a. Determine the sag at B, the tension in the cable, and the length of the cable.Answer:



Support reactions. The reactions of the cable are determined by applying the equations of equilibrium to the free-body diagram of the cable shown in Figure 6.8b, which is written as follows:

$$+\curvearrowleft \sum M_A = 0$$
$$-100(2) - 80(10) + 13D_y = 0$$
$$D_y = 76.92 \text{ kN}$$

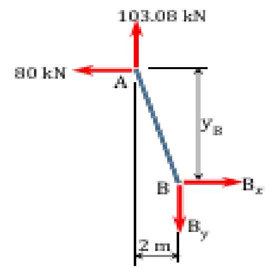
$$+\uparrow \sum F_y = 0$$

$$A_y + 76.92 - 100 - 80 = 0$$
$$A_y = 103.08 \text{ kN}$$

$$+\curvearrowleft \sum M_C = 0$$
$$-A_x(10) + 100(8) = 0$$
$$A_x = 80 \text{ kN}$$

$$+\rightarrow \sum F_x = 0$$
$$-D_x + 80 = 0$$
$$D_x = 80 \text{ kN}$$

Sag at B. The sag at point B of the cable is determined by taking the moment about B, as shown in the free-body diagram in Figure 6.8c, which is written as follows:



(c)

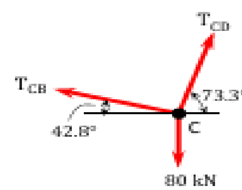
$$+\curvearrowleft \sum M_B = 0$$
$$-A_y(2) + A_x(y_B) = 0$$
$$y_B = \frac{A_y(2)}{A_x} = \frac{103.08(2)}{80} = 2.58 \text{ m} \qquad y_B = 2.58 \text{ m}$$

Tension in cable.

Tension at A and D.

$$T_A = T_{AB} = \sqrt{(A_y)^2 + (A_x)^2} = \sqrt{(103.08)^2 + (80)^2} = 130.48 \text{ kN}$$

$$T_D = T_{DC} = \sqrt{(D_y)^2 + (D_x)^2} = \sqrt{(76.92)^2 + (80)^2} = 110.98 \text{ kN}$$



(d)

Tension in segment CB.

$$+\rightarrow \sum F_x = 0$$
$$T_{CD}\cos 73.3° - T_{CB}\cos 42.8° = 0$$
$$T_{CB} = \frac{T_{CD}\cos(73.3°)}{\cos 42.8} = \frac{110.98\cos(73.3°)}{\cos 42.8} = 43.46 \text{ kN}$$

Length of cable. The length of the cable is determined as the algebraic sum of the lengths of the segments. The lengths of the segments can be obtained by the application of the Pythagoras theorem, as follows:

$$L = \sqrt{(2.58)^2 + (2)^2} + \sqrt{(10 - 2.58)^2 + (8)^2} + \sqrt{(10)^2 + (3)^2} = 24.62 \text{ m}$$

Q8. Explain analysis of cables subjected to uniformly distributed load (UDL).

Answer:

Consider a free-body diagram of a segment of the cable extending from the lowest point, C, to an arbitrary point, P(x,y), as shown in the image below.

1. Forces Acting on the Segment CP

The segment CP is subjected to three forces:

- **Horizontal Tension (H)** at the lowest point, C. Since the tangent is horizontal at C, the tension is purely horizontal.
- **Tension (T)** at point P, acting tangentially to the cable at an angle θ with the horizontal.
- **Resultant Load (W)** due to the UDL. Since the load is w per unit *horizontal* length and the horizontal projection of CP is x, the total load is:

  $W=w\cdot x$

  This resultant load acts vertically downward at the centroid of the load distribution, which is at a horizontal distance of x/2 from P.

. Equilibrium Equations

For the segment CP to be in equilibrium, the sum of horizontal and vertical forces must be zero:

- **Horizontal Equilibrium (∑Fx=0):** The horizontal component of the tension at P must equal the horizontal tension at C.

  $T\cos\theta-H=0$

  $T\cos\theta=H(i)$

- **Vertical Equilibrium (∑Fy=0):** The vertical component of the tension at P must balance the total vertical load W.

  $T\sin\theta-W=0$

  $T\sin\theta=W=wx(ii)$

. Determining the Cable Shape (Equation of the Curve)

The slope of the cable at point P is given by tanθ:

$dxdy=\tan\theta$

Divide equation (ii) by equation (i):

$T\cos\theta T\sin\theta=Hwx$

$\tan\theta=Hwx$

Substitute tanθ with dy/dx:

$dxdy=Hwx$

Integrate this first-order differential equation with respect to x to find y:

$\int dy=\int Hwxdxy=Hw\int xdxy=2Hwx2+C1$

where C1 is the constant of integration.

. Applying Boundary Condition

To find the constant C1, we use the boundary condition that at the lowest point, C, the coordinates are (0,0). At x=0, y=0:

$0=2Hw(0)2+C1$

$C1=0$

Substituting C1=0 back into the equation for y:

$y=2Hwx2$

This is the standard equation of a **parabola** with its vertex at the origin (0,0).

Maximum and Minimum Tension

- **Minimum Tension:** Occurs at the lowest point, C, where the tangent is horizontal.

  $Tmin=H$

- **Maximum Tension:** Occurs at the supports (points furthest from the minimum sag, y). The tension at any point P is found by squaring and adding equations (i) and (ii):

  $T2cos2\theta+T2sin2\theta=H2+(wx)2$

  $T2(cos2\theta+sin2\theta)=H2+(wx)2$

  $T=H2+(wx)2$

  Since w, H, and x2 are all positive, **T is maximum when x is maximum** (i.e., at the supports).

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Q9. Discuss the effect of temperature changes on cables and derive expression for change in length.

Answer:

The primary effect of temperature on a cable's physical dimension is **linear thermal expansion**.

- **Heating (Temperature Increase):** When a cable's temperature increases (due to high ambient temperature or heat generated by the current passing through it), the material's constituent atoms vibrate more vigorously. This increased vibration forces the atoms further apart, causing the cable to **expand** and its **length to increase**. In overhead cables, this causes increased **sag**.
- **Cooling (Temperature Decrease):** Conversely, when the temperature drops, the atomic vibrations lessen, allowing the atoms to move closer together. This causes the cable to **contract** and its **length to decrease**. This can lead to increased **tension** and potentially dangerous stress on the cable and its supports.

Other Related Effects

While the physical change in length (thermal expansion) is the direct result, temperature also affects other cable properties, especially for electrical cables:

- **Electrical Resistance:** For most conductor materials (like copper and aluminum), resistance increases with temperature. This is because the more vigorous atomic vibrations impede the flow of electrons.
- **Insulation Performance:** High temperatures can damage or degrade the cable's insulation and jacket, reducing their effectiveness and lifespan. Extremely low temperatures can cause some insulation materials to become brittle and crack.
- **Current Carrying Capacity (Ampacity):** Due to the increase in resistance at higher temperatures, the maximum current a cable can safely carry (its ampacity) is reduced to prevent overheating and thermal damage.

Derivation of Expression for Change in Length

The change in the linear dimension (length) of a solid due to a change in temperature is described by the formula for **linear thermal expansion**.

1. The Phenomenon

The change in length (ΔL) is directly proportional to three factors:

1. The **original length** ($L0$).
2. The **change in temperature** (ΔT).
3. The material's specific property called the **coefficient of linear thermal expansion** (α).

Mathematically, this proportionality can be written as:

$$\Delta L \propto L0 \cdot \Delta T$$

2. Introducing the Coefficient of Linear Thermal Expansion (α)

To turn the proportionality into an equality, we introduce the constant of proportionality, α:

$\Delta L=\alpha \cdot L0 \cdot \Delta T$

This is the standard expression for the change in length.

3. The Final Length

The **final length** (L) of the cable after the temperature change is the original length plus the change in length:

$L=L0+\Delta L$

Substituting the expression for ΔL:

$L=L0+\alpha \cdot L0 \cdot \Delta T$

Factoring out the original length (L0):

$L=L0(1+\alpha \cdot \Delta T)$

Q10. A cable of span l has its heights h1 and h2 above the lowest point of the cable. It carries a UDL of w per unit run of the span. Determine the vertical reactions at each end..

Answer:

The vertical reactions at each end of the cable are calculated based on the principles of static equilibrium for a simply supported beam with a uniformly distributed load (UDL). Although the cable has a sag, the vertical reactions are determined by considering the total vertical load and its distribution.

Vertical Reactions

For a cable (or beam) of span l carrying a uniformly distributed load of w per unit run over the entire span, the total vertical load is $W=w \cdot l$.

Since the load is **uniformly distributed** over the entire span l, the problem is **symmetrical** with respect to the vertical reactions, even if the cable heights h1 and h2 are different, because the load itself is symmetric.

The vertical reaction at each end, VA and VB, is half of the total load.

$VA=VB=2W$

Substituting the total load $W = w \cdot l$:
$VA=VB=2w \cdot l$

The heights h1 and h2 are relevant for determining the **horizontal tension component** and the maximum tension in the cable, but they **do not affect the calculation of the vertical reactions**. The vertical reactions only depend on the total vertical load carried by the span.

🎬 Let the lowest point of the cable be the origin $x = 0$. Let the left support be at $x = -x_1$ and the right support at $x = +x_2$. The horizontal span is
$l = x_1 + x_2$.

🎬 The total resultant vertical load from the uniform load w (per unit horizontal length) is
$W = w \times l$.

🎬 For a load uniformly distributed over the horizontal interval $[-x_1, x_2]$, the resultant acts at the midpoint of that interval. Its horizontal location measured from the left support ($x = -x_1$) is
$d = l / 2$.

🎬 Take moments about the left support: the moment of the applied load about the left support is $W \times d = w \times l \times (l / 2)$. That must be balanced by the right vertical reaction R_2 producing moment $R_2 \times l$. Thus
$R_2 \times l = W \times (l / 2) \rightarrow R_2 = W / 2 = w \times l / 2$.

🎬 Vertical equilibrium gives $R_1 + R_2 = W$, so $R_1 = W - R_2 = W / 2$ as well.